MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words _____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

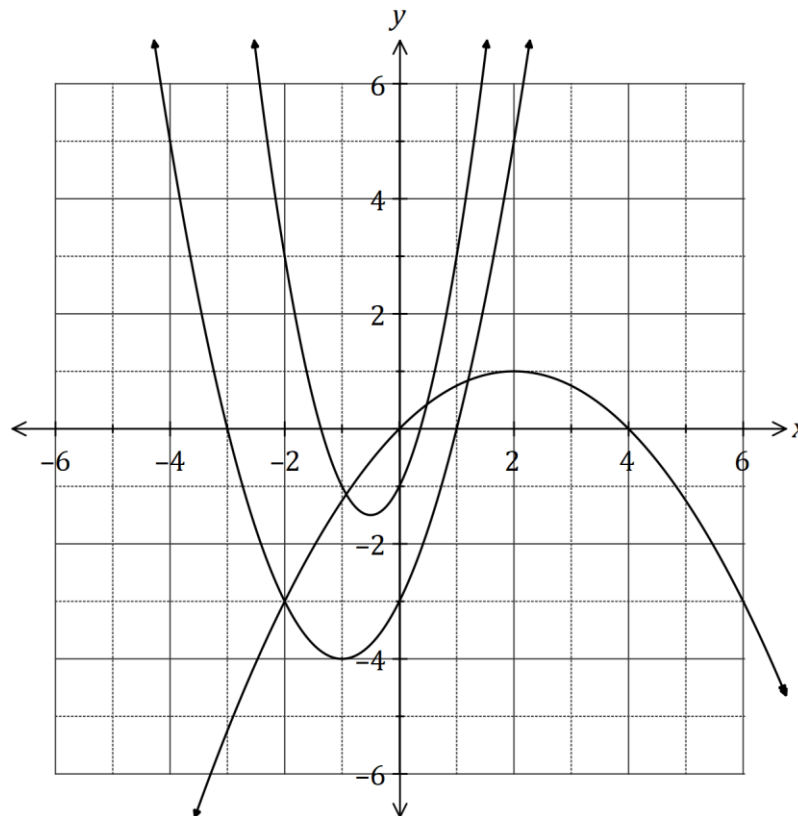**Section Two: Calculator-assumed** 65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (4 marks)

The graphs of $y = 2x^2 + 2x + c$, $y = a(x - 2)^2 + 1$ and $y = (x + b)(x + 3)$ are shown below.

Determine the values of the constants a, b and c.

See next page

Question 10 **(7 marks)**

An online cosmetics company is offering new customers the opportunity to select 6 different products for just $6 each. They can select from a range of 11 different moisturisers, 15 different lipsticks and 9 different eye-shadows.

(a) Determine how many different selections can be made. (2 marks)

(b) Determine how many different selections can be made that just include moisturisers.
(2 marks)

(c) Determine the probability that if a new customer randomly makes a selection, it will not include an eye-shadow. (3 marks)

Question 11 **(6 marks)**

A thin pole stands vertically in the middle of a level playing ground. From point A on the ground, the angle of elevation to the top of the pole, T, is 18^0.

From point B, also on the ground but 5.35 metres further from the foot of the pole than A, the angle of elevation to the top of the pole is 15^0.

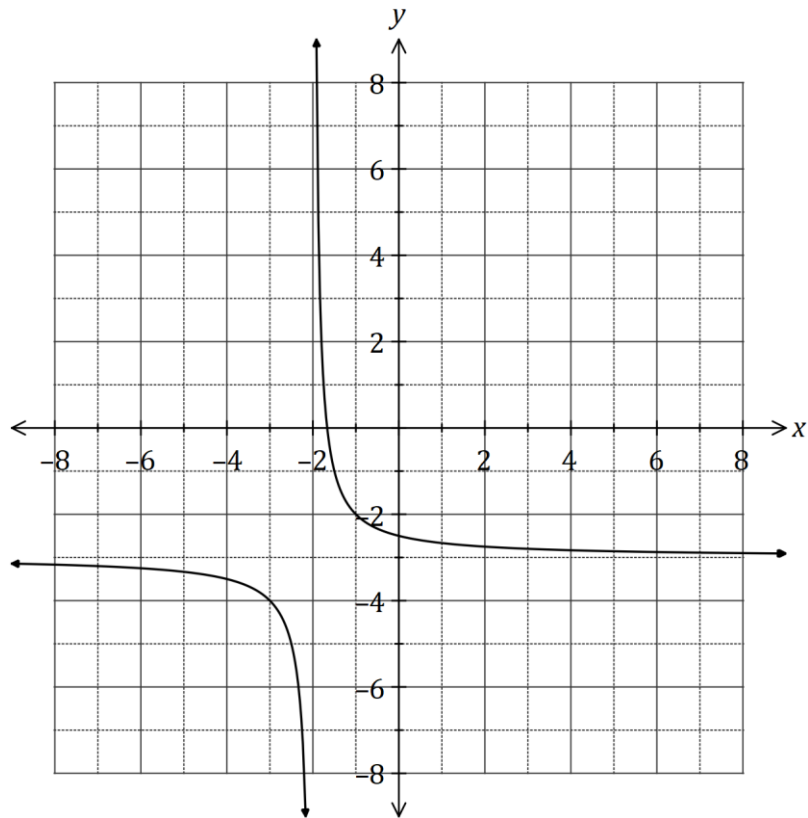(a)   Draw a sketch to represent this information.   (1 mark)

(b)   Showing use of trigonometry, determine the height of the pole.   (5 marks)

Question 12   **(7 marks)**

Let $f(x) = \dfrac{4}{3 - x}$ and $g(x) = \dfrac{1}{x + p} + q$, where p and q are constants.

The graph of $y = g(x)$ is shown below.

(a)   Sketch the graph of $y = f(x)$ on the axes above.   (3 marks)

(b)   Determine the values of p and q.   (2 marks)

(c)   Solve the equation $f(x) = g(x)$, giving your solution(s) to one decimal place.   (2 marks)
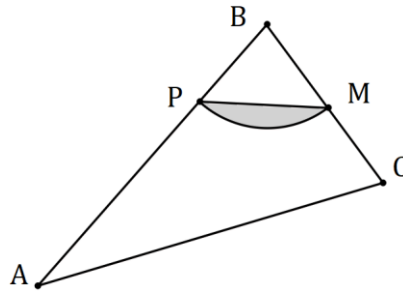
Question 13 **(6 marks)**

(a) Determine the equation of the axis of symmetry for the graph of $y = 3x^2 + 12x + 40$.

(2 marks)

(b) The graph of $y = ax^2 + bx + 13$ passes through the points $(-3, -23)$ and $(4, 5)$.
Determine the values of the constants a and b. (4 marks)

Question 14 **(10 marks)**

A logo with triangular outline ABC contains a shaded segment bounded by the straight line PM and the circular arc PM with centre B and radius $BM = 18$ cm, as shown below.

Given that $\angle ABC = \dfrac{5\pi}{12}$, $\angle BCA = 2\angle BAC$ and M is the midpoint of BC, determine

(a)   the size of $\angle ABC$ in degrees. (1 mark)

(b)   the area of the shaded segment. (2 marks)

(c)   the perimeter of the shaded segment. (3 marks)

(d)   the area of triangle ABC.   (4 marks)

Question 15 **(9 marks)**

In a group of 150 students, it is known that 40% are in Year 11 and the rest in Year 12. 60% of those in Year 11 and 50% of those in Year 12 study maths, while the remainder do not.

Let E be the subset of students in Year 11 and M be the subset of students who study maths.

(a) Use set language and notation to describe the subset of students who

   (i) are in Year 12. (1 mark)

   (ii) are in Year 11 or study maths. (1 mark)

(b) Determine the probability that a randomly chosen student from the group

   (i) is in Year 12 and studies math. (1 mark)

   (ii) studies math. (2 marks)

   (iii) is in Year 12, given that they study math. (2 marks)

   (iv) is in Year 11 or studies math. (2 marks)

Question 16 **(6 marks)**

(a) The graph of $y = f(x)$ is shown in bold below. The graphs of $y = -f(x)$, $y = f(x + p)$ and $y = f(x) + q$ are also shown, where p and q are constants.
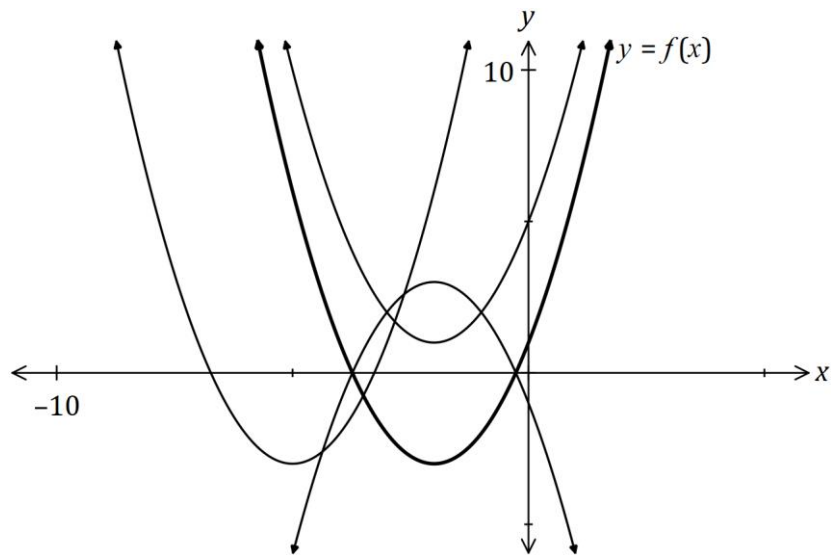
Clearly label the remaining graphs with $y = -f(x), y = f(x + p)$ or $y = f(x) + q$.

(3 marks)

(b) The one-to-one relation $y = 7 - 3x$ has domain and range given by $\{x: x = -2, 3, a\}$ and $\{y: y = -8, -2, b\}$ respectively. Determine the values of constants a and b. (3 marks)

Question 17 **(9 marks)**

For two events, X and Y, it is known that $P(X) = 0.64$ and $P(Y) = 0.55$. Determine the following probabilities, using any additional information only within that part of the question.

(a) $P(\bar{X})$. (1 mark)

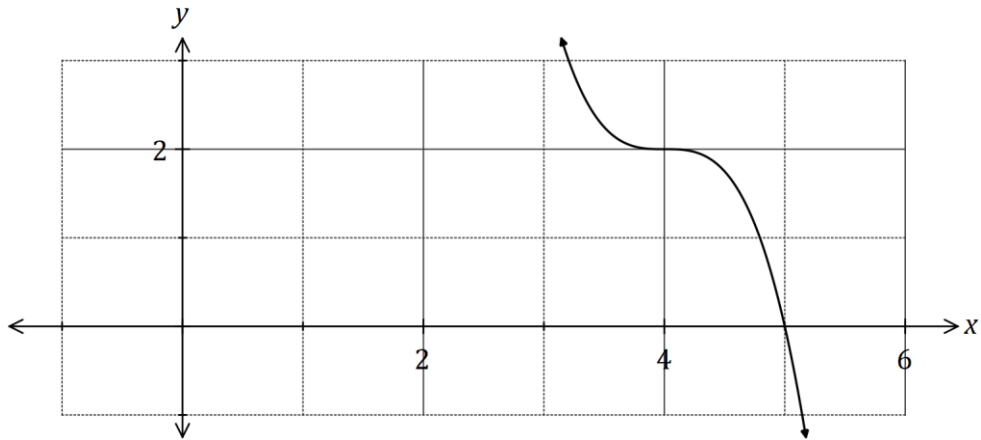(b) $P(X \cap \bar{Y})$ when $P(Y \cap \bar{X}) = 0.18$. (2 marks)

(c) $P(X \cup Y)$ when X and Y are independent. (3 marks)

(d) $P(Y|X)$ when $P(X|Y) = 0.96$. (3 marks)

Question 18 (6 marks)

(a) Part of the graph of $y = f(x)$ is shown below, where $f(x) = -2(x-b)^3 + c$, and b and c are constants.

(i) State the degree of $f(x)$. (1 mark)

(ii) Determine the value of b. (1 mark)

(iii) Determine $f(0)$. (2 marks)

(b) Another function is given by $g(x) = f(x+8)$.

Describe how to obtain the graph of $y = g(x)$ from the graph of $y = f(x)$. (2 marks)

Question 19 **(12 marks)**

During 2018, the altitude of the sun, θ degrees, at noon in Melbourne on the nth day of the year can be modelled by the equation

$$\theta = 23.5 \sin\left(\frac{8\pi(101 + n)}{1461}\right) + 52.2$$

(a) On the 26th of January, the altitude was 71.4°. Calculate the altitude ten days earlier.

(2 marks)

(b) Graph the altitude on the axes below for $0 \le n \le 365$. (4 marks)
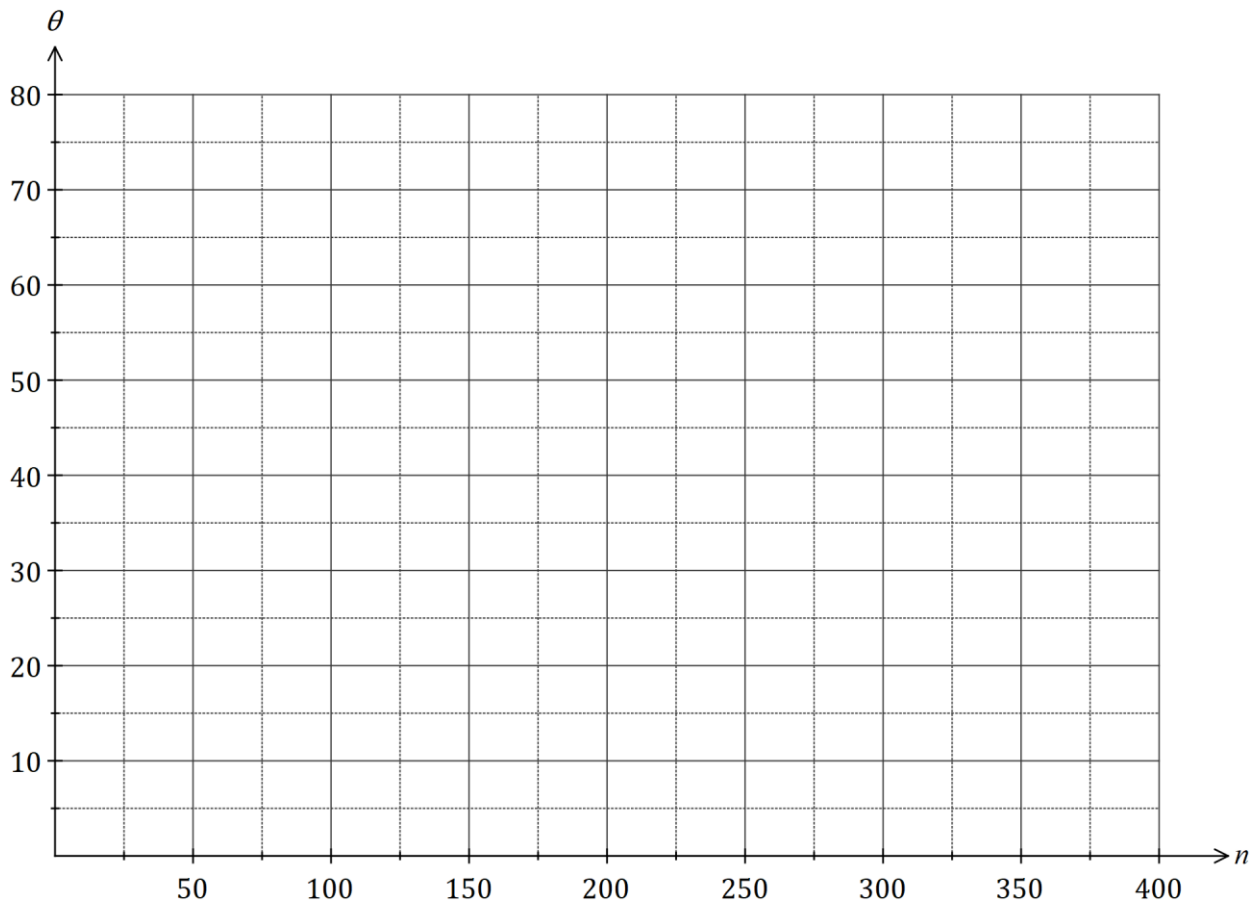
(c)   State the minimum altitude of the sun at noon in Melbourne and on which day of the year this occurred. (2 marks)

(d)   Solar panels on the roof of a Melbourne business are designed to meet its entire power needs on cloudless days when the altitude of the sun is at least 36° at noon.

   (i)   Draw a straight line on your graph to represent this requirement. (1 mark)

   (ii)   Determine the number of days the panels are expected to achieve this aim during 2018, ignoring the possibility of cloud cover. (3 marks)

Question 20 **(7 marks)**

Consider the points with coordinates (p, q) and (r, s) that lie in the first and second quadrants respectively of the unit circles shown below, where θ and γ are acute angles.
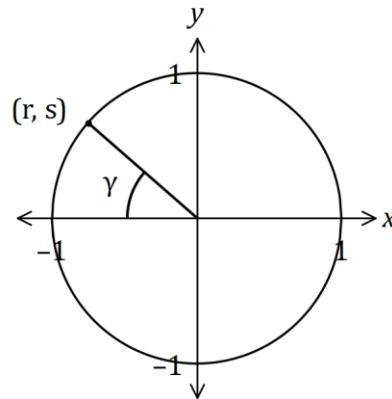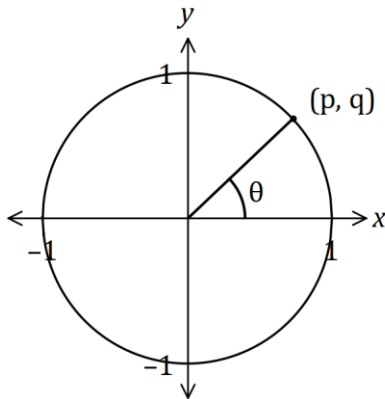
Determine the following in terms of p, q, r and s, simplifying your answers where possible.

(a) $\tan \theta$.   (1 mark)

(b) $\sin(180° - \theta)$.   (1 mark)

(c) $\cos \gamma$.   (1 mark)

(d) $\sin(\pi + \gamma)$.   (1 mark)

(e) $\cos(\gamma - \theta)$.   (3 marks)

Question 21 **(9 marks)**

Injury events in a state were grouped as follows: (D) injury events resulting in death; (H) injury events requiring hospitalisation; and (E) injury events requiring treatment at an emergency department. Injury events were further grouped according to whether prior use of alcohol was a contributing factor. The number of injury events for one month is summarised in the table below.

	D	H	E	Total
Alcohol	25	581	5 605	6 211
No Alcohol	115	4 422	11 912	16 449
Total	140	5 003	17 517	22 660

(a)   Calculate the percentage of all injury events that involved prior use of alcohol.   (2 marks)

(b)   If one injury event was selected at random from the data, determine the probability that

(i)   it required hospitalisation.   (1 mark)

(ii)   it did not require treatment at an emergency department given that prior use of alcohol was a contributing factor.   (2 marks)

(iii)   prior use of alcohol was a contributing factor given that it required hospitalisation.   (2 marks)

(c)   Use your answers above to explain whether the type of injury event (D, H or E) appears to be independent of prior use of alcohol.   (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____